Review of *Nonnegative Matrices*
By Henryk Minc*

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This book is intended as an introduction to the theory of nonnegative matrices and as a self-contained reference work for scientists and mathematicians. Nonnegative matrices arise in applications in probability theory, economics, numerical analysis, and the theory of small oscillations of elastic systems, to name just a few areas. In this book such matrices are studied primarily to determine their algebraic properties.

The book has seven chapters. The first three chapters develop the basic Perron-Frobenius theory following the method developed by Wielandt [1]. The bounds due to Ledermann, Ostrowski, and Brauer for the maximal eigenvalue of a positive matrix are proved in Chapter II.

Chapter IV deals with structural properties of nonnegative matrices. By this is meant properties of nonnegative matrices that depend only on the pattern of zeros. Covered here are the Frobenius-König theorem (stated as a result about the permanent of a nonnegative matrix), fully indecomposable matrices, Hartfiel's proof of a canonical form for nearly decomposable and nearly reducible matrices, bounds for the number of 1's in a nearly decomposable $(0,1)$ matrix, and bounds for permanents of $(0,1)$ matrices, including Schrijver's elegant proof of the Minc conjecture on the upper bound for the permanent of a $(0,1)$ matrix.

Chapter V deals with properties of doubly stochastic matrices. As expected, the Birkhoff theorem is proved along with the Hardy-Littlewood-Polya generalization of Muirhead's theorem. There is a discussion of Farahat and Mirsky's problem of determining for a given doubly stochastic matrix A the least number of permutation matrices whose convex combination is A. The

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chapter concludes with a proof of the van der Waerden conjecture, following Falikman and Egoryčev.

Chapter VI deals with other classes of nonnegative matrices. These include stochastic matrices, totally nonnegative matrices, oscillatory matrices, and M-matrices. The viewpoint of the chapter is that it is sufficient to prove properties of these matrices. I think some motivation by way of example would have helped to orient the reader to why these particular matrices are important.

Chapter VII provides an introduction to the inverse eigenvalue problem for nonnegative (or stochastic or doubly stochastic) matrices. The problem was completely solved by Karpelevič [3] for stochastic matrices. His results are stated, but no proof is provided. The doubly stochastic case is an interesting unsolved problem. I would like to have seen a reference at least for this problem to [5], which has a large collection of references to this subject, and in particular leads to [6], where the $n = 3$ case is solved. There is also an introduction to the inverse-spectrum problem for nonnegative matrices, where the results of Loewy and London (necessary conditions) are given, and to the inverse-elementary-divisor problem for nonnegative (or doubly stochastic) matrices.

As a textbook I think this book has an interesting selection of topics with a good collection of worked examples and exercises (154 in all) of varying difficulty. I would expect it would work well for an upper-division course in nonnegative matrices. It would be improved by additional motivation for some of the topics, particularly in Chapter VI, and some bibliographic notes at the end of each chapter. Also, the author sometimes seems to favor providing his own proof of a result, rather than a simpler proof that has appeared in the literature. For example, Pullman's proof [4] of the theorem in Chapter 3 that if the product of the nonzero blocks of a matrix in superdiagonal form is irreducible, then the matrix itself is irreducible seems much shorter and simpler than the one the author gives. As another example, the existence of the Frobenius normal form for an irreducible matrix of index $h \geqslant 2$ is proved by relying on detailed knowledge of the eigenvalues of the matrix, as was done by Wielandt [1]. A much simpler approach is to follow the proof given by Dulmage and Mendelsohn [7] using the theory of cyclically k-partite directed graphs.

The choice of topics seems to heavily reflect the author's own research interests, and as a result the range and coverage is not as comprehensive as one would like to see in a reference book. Indications of this emphasis are the set of references: the book has only 92 references to other books and papers, compared to over 360 in Berman and Plemmon's [2] work. Moreover, there are only seven references to papers since 1981, and three of these are on the van der Waerden permanent problem, and four to Minc's own work.

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